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Optimisation of Railway Ticketing System during National Day based on Queuing Theory

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Abstract

In recent years, with the prosperous development of China's railway industry, passengers' needs for the railway system have also been refined, focusing on the basic experience while paying more attention to the issue of travel efficiency. Window ticketing is the most reliable way to purchase tickets as a countermeasure in case of network system and call system crashes during heavy traffic, so a reasonable number of ticketing windows is important to improve ticketing efficiency. Too few ticket windows can lead to long queues during peak periods, creating congestion problems and affecting the operation of the railway system. Too many ticket windows can lead to a waste of manpower, electricity and other related resources at stations during periods of low passenger traffic, increasing the operating costs of the railway system. Arranging ticketing windows according to demand, such as adding a certain number of windows during peak periods and closing some windows during periods of low passenger traffic, allows the station's various resources to be fully utilised to meet the ticketing needs of passengers at different times, thus improving the overall efficiency of the ticketing process. Therefore, the key to this paper is to address the issue of how to reasonably set the number of windows for ticketing in different situations in relation to the field situation of the station and the queuing behaviour of passengers. In this paper, the real-time passenger traffic data of Hefei South Railway Station for each month from 2013 to 2016 were selected as the basic data, and the passenger flow data and window service time data of the selected peak periods were probability sampled, and the distribution of passenger flow and window service time were obtained by using the maximum likelihood estimation. The estimated values of the population parameter are $\lambda=5$ (person/min) and $\mu=1.6$ (min/person) respectively. The two distributions were verified as possion distribution and negative exponential distribution, and the optimum number of windows to be opened at the station is 6-7 by simulation, taking into account the maximum tolerance time of the passengers studied. According to the analytical ideas in this paper, in terms of the future opening of railway ticket windows, the size of passenger flow and service efficiency are reasonably estimated, and the number of open windows is adjusted according to peak and low passenger flow in order to save manpower, coordinate passengers and maximise resource utilisation.

Keywords

Queuing Theory; Railway Ticketing System; Optimisation.

1. Introduction

1.1. Research Background

Railways are an important national infrastructure playing an important role in promoting theprocess of economic and social development in China, not only as the main medium for transporting goods, but also as a mass transportation, with the continuous development of

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China's high-speed railway business, the speed, convenience and comfort of the railway system have made it the primary choice of the public when travelling. In the process of using the railway system, the first issue facing passengers is ticketing, which is not only related to the passengers themselves, but also reflects the interaction process between the passengers and the railway authorities, so improving the process of ticketing for passengers, bringing them a good experience and improving the efficiency of the ticketing process is one of the priorities of the railway system.

According to China Railway Network, there are six ways to purchase tickets in China's railway system: ticket window purchase, automatic ticketing machine purchase, telephone booking, Internet purchase, standby purchase, and contact reservation. In non-holiday periods and some special time periods, these ways of purchasing tickets can be chosen by passengers at will according to their own situation, but due to the limited service capacity of some systems in the railway system, when the network and telephone systems are congested and However, due to the limited service capacity of some systems in the railway system, when the network and telephone systems are congested and malfunctioning, people tend to choose the time-consuming and non-self-operating way - window ticketing, so reasonable arrangements for setting up railway ticketing windows are the root cause of the ticketing problem.

The numbers of the ticket window varies depending on the station's geographical location, passenger flow and other basic conditions, and the different setting sizes may lead to the following three situations:

(1) The number of ticketing windows is too small: especially during peak ticketing periods such as legal holidays and students' summer and winter vacations, too few windows will make the queue more crowded, causing the station to be overcrowded and unable to meet passengers' ticketing needs in a timely manner, affecting passengers' travel plans making them negative towards the railway system, leading to inefficient ticketing relief and affecting the process of subsequent links.

(2) The number of ticketing windows is excessive: when the passenger flow is normal or low it will lead to some windows being left idle, causing a certain degree of waste of human, financial and electrical resources of the station.

(3) Adjustment of the number of ticket windows according to needs: In this case, a certain number of new windows are added when the passenger flow is high, which can ease the pressure on passengers and stations to purchase tickets; some windows are closed when the passenger flow is low, so that all kinds of resources are not wasted and can be fully utilised; reasonable adjustments are made according to the actual situation on weekdays, and such settings can significantly improve the efficiency of the railway system's ticketing process.

1.2. Research Content

In this paper, an overview of the $M/M/c/\infty$ model is given first. After understanding the theoretical basis for the meaning of the parameters of the model and its mathematical derivation, the real-time data set of passenger flow at Hefei South Station is collected as the base data, and the distribution of the data to be studied is determined and incorporated into the model based on a corresponding understanding of the relevant Poisson distribution and negative exponential distribution. The parameters required for the model are also calculated. Finally, the problem of opening the ticket window at the railway window during peak hours is optimised through simulations and by combining the parameters deduced from the data. As the aim of this paper is to meet the needs of passengers as much as possible within the cost of the railway sector, the longest waiting time that passengers can endure in the queue is taken as an important factor in determining whether the number of windows is reasonable.

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1.3. Introduction to the M/M/C/∞ Model

Queuing Theory, also known as stochastic service system theory, is a sub-discipline of operations research, which focuses on the impact of stochastic factors on the overall system and the stochastic nature of the waiting queue in various service systems through the analysis of several aspects of the nature problem, statistical inference, and optimization problems. The model states that the arrival time of customers obeys a negative exponential distribution, the service time of service stations also obeys a negative exponential distribution, there are c service stations providing service, and the service rule is a first-come, first-served rule. When a customer arrives at the queuing system, the customer is judged to be able to accept the service according to the situation at the service desk. When a service desk is free, i.e. the number of customers in the queuing system is less than the number of service desks, the customer can accept the service immediately. If a customer arrives at the queuing system and no service desk is available, the customer will need to enter the queue and wait until the previous customer has received the service, i.e. when another service desk is available. It can be assumed that the time interval between customers arriving at the queuing system is random, obeying the Poisson flow which takes in as a parameter, while the time when each customer receives service is also random, and the service times are independent of each other, obeying the negative exponential distribution which takes μ as a parameter. The system can be seen as a model of multi servers and single queue.

In this model of a queuing system, the following mathematical calculations can be made, resulting in several parameters as follows:

$$\text{Make } \rho = \frac{\lambda}{\mu}, \rho_c = \frac{\lambda}{c\mu} \tag{1}$$

$$\text{Let } \rho_j = \lim_{t\to\infty} P\{N_{(t)} = j\}, j = 0,1,\cdots, \tag{2}$$

then when $\rho_c < 1$, there exists$\{\rho_j, j \geq 0\}$, independent of the initial condition, and where

$$P_0 = \left[\sum_{j=0}^{c=1} \frac{\rho_j}{j!} + \frac{c\rho^t}{c!(c-\rho)}\right]^{-1} \tag{3}$$

Service intensity (ρ) is an indication of the efficiency of service at the railway ticket window. Service intensity is calculated as the average arrival rate of passengers going to a ticket window on a train λ compared to the average service rate of ticket windows across the system $c\mu$ i.e. $\rho = \frac{\lambda}{c\mu}$, $(0 < \rho < 1)$ which is the number of passengers purchasing tickets in a given period of time and the number of passengers that can be received.

The ratio between the services received. The average length of stay (W_s) is the amount of time a ticketed passenger spends in the ticketing system.

$$W_s = \frac{L_s}{\lambda} \tag{4}$$

The average queue length (L_s) is the number of people waiting to be served and being served in the queue system as a whole for the queue length (L_q), and in general, the larger their values are, the more inefficient the overall queue system is in terms of service, whether it is the average queue length or the queue length.

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2. Literature References

Queuing theory was founded by Danish mathematician Erlang and has been developed over the years. The queuing theory is based on the collection of data from the system to be studied, the analysis of the service targets, service times and the allocation of resources in the queuing system, and the establishment of relevant mathematical models, which are used to obtain quantitative index laws and then improve the configuration of the service system or reorganise the resources of the service according to these laws, so that the relevant system can meet the needs of the service targets while minimising its own service costs. The system can meet the needs of the service users and minimise the cost of the service, thus making the service system optimal [1]. In everyday life, queuing problems are often caused by a lack of resources, so many managers need to use queuing theory to find an optimal allocation solution that maximises benefits without losing users [2]. The purpose of solving queuing problems is therefore to study the efficiency of queuing system operation, assess the quality of service, determine the optimal value of system parameters in order to understand whether the system structure is reasonable, study the design of improvement measures, etc[3]. Queuing theory not only allows for the rational design and effective management of a service system so that the service system can meet the needs of the passengers, but also allows for the most economical cost dissipation or optimal service indicators of the service system[4].

The development of queuing theory also brings help to different fields to solve practical problems. Based on the queuing model, Yifei Zhao and Xiaoying Qiao built an M/G/1 queuing model for general service hours thus predicting the approach time of flights in the terminal area, which reduced the difficulty of existing models and also brought guiding suggestions for improving flight operation control[5]. Xintao Guo and Jun Zhang et al. constructed a tandem multi-service desk single queue queuing model by counting the relevant record data of a certain 10 vaccination clinics in Suzhou City to adjust the operation index of each link to improve the efficiency of vaccination clinics and alleviate the crowded and disorderly situation of clinics[6]. Jirong Yao and Dechang Zhao used the taxi airport queuing to pick up passengers as a background and combined the M/M/1 classical queuing theory model to establish a new cost-income model, which comprehensively analysed the impact of different factors on drivers' revenue, enabling drivers to make decisions that make their personal revenue optimal in different scenarios[7]. Based on the principle of queuing theory, Han Snap, Duolong Wang et al. optimised the layout and configuration of security screening equipment in a metro station to reduce the queuing time and improve the efficiency of security screening. Kexue Jin, Wenjing Zhao et al. investigated a bank's branch business outlets, established a bank queuing model, simplified and analysed the existing M/M/c system model, and concluded that the establishment of four service windows in this business outlet is the best way to reduce the queuing time of customers and improve their satisfaction, while enhancing the competitiveness of the bank, and proposed measures to improve the efficiency of bank services from other perspectives[8].

3. Data Processing

As a hub for inland passage, Hefei South Station has a strategic position in connecting the resources and logistics of the Upper Yangtze River Delta region. In this paper, the passenger flow of Hefei South Station is selected as the source of data, firstly, we collected the data on the passenger volume of each month of the station from 2013 to 2016, as shown in Figure 1.

Peak passenger flow refers to a large movement of people within a short period of time, resulting in a certain amount of time waiting in line at the station for ticketing rides. Taking November as an example, as a statutory public holiday in China, a large number of passengers

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visit for a trip every year on November. We have collected the passenger flow for each year from 2013-2016 for the November holiday, as shown in Figure 2.

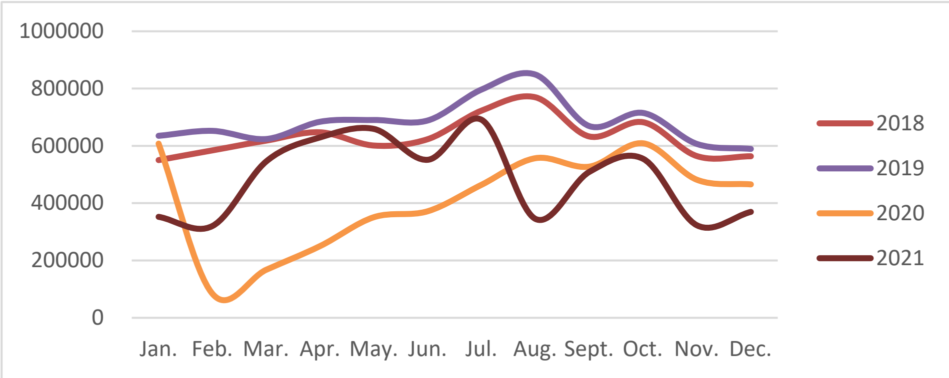

Figure 1. Passenger traffic by month from 2018 to 2021

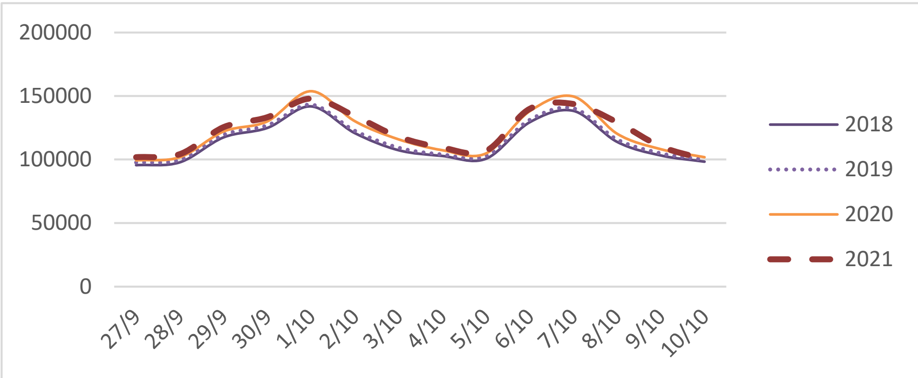

Figure 2. Passenger traffic during the November holidays from 2018 to 2021

In order to verify the specific distribution of passenger flow during the peak period, we selected the two days of October 1 and the rest of the weekdays as the base data and conducted statistics by time period, the results are shown in the Table 1 below.

Table 1. Peak and off-peak passenger arrival rates by time of day (person/h)

Time	Peak period	Off-peak period
0-1	250	180
1-2	174	212
2-3	161	217
3-4	153	217
4-5	169	230
5-6	268	276
6-7	485	567
7-8	1705	862
8-9	1996	855
9-10	1790	778
10-11	1542	951
11-12	1627	996
12-13	1728	1047

13-14	1577	1038
14-15	1037	1083
15-16	1222	932
16-17	874	978
17-18	950	999
18-19	813	1022
19-20	646	708
20-21	311	496
21-22	331	270
22-23	286	193
23-24	357	180

According to the above table we can find that the ticketing system is mainly concentrated between 7:00am and 19:00pm, which is in line with the objective facts. In this paper, in order to explore the general pattern, the next analysis mainly focuses on the daytime time period, and the higher number of passenger data sets during the day also reduces certain analysis bias.

4. Optimization Model of Railway Ticketing System based on Queuing Theory

4.1. Determination of the Passenger Flow Distribution

The determination of the passenger flow distribution involves nonparametric statistics, our main approach is to take one of the hourly passenger flows for probability statistics, in an arbitrary time period from this hour, randomly selected 100 five-minute time periods, and then observe the passenger flow in each five-minute time period under these 100 time periods. The frequency histogram is equivalent to the 100 times sampling, and the frequency histogram is equivalent to the probability density.

Before conducting the non-parametric test, we need to determine the original hypothesis H0 and the alternative hypothesis H1 .

H0: Distribution of passenger arrivals as non-possion distribution

H1: Distribution of passenger arrivals as possion distribution

The maximum likelihood estimate of λ for the parameters of the Poisson distribution is calculated as follows.

$$\hat{\lambda} = \frac{1}{n}\sum_{i=1}^{n} X_i = \overline{X} \tag{5}$$

Table 2 shows that the number of passenger arrivals between 7am and 8am during the off-peak period is 862, i.e. the passenger arrival rate$\lambda = \overline{X} = 71.8$. Divide the number of passenger arrivals per unit time X in the histogram into five groups [0,64], [65,74], [75,84], [85,94], [95,+∞] and calculate the probability $p_i(X = i)$

Where a_{n-1} is the lower limit of each set of intervals and a_n is the upper limit of each set of intervals.

We carried out the distribution test along the lines described above, with the following procedure.

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Table 2. The table of distribution test

i	f_i	p_i	np_i	$\frac{(f_i - np_i)^2}{np_i}$
1	14	0.1555	15.55	0.1545
2	44	0.4519	45.19	0.0313
3	31	0.3249	32.49	0.0683
4	10	0.0638	6.38	2.0540
5	1	0.0039	0.39	0.9541
Σ	100			3.2622

Based on the results of the above distribution test, the chi-squared value is 3.2622, which rejects the original hypothesis at the α=0.05 level of significance, i.e. the distribution of passenger arrivals is the possion distribution.

4.2. Determination of Service Time Distribution

Since the determination of service hours is subjective and there are large individual differences depending on the time of day and the conductor, the service efficiency is divided into two categories u_1 , u_2 , with the more efficient u1 used during peak periods and the less efficient u_2 used during off-peak periods.

The passenger flow between 7-8pm on 1 October and the rest of the weekdays was selected as the base data, and the statistics were divided into time periods. Our main approach was to select the information of all tickets sold between 7-8pm during the peak period as the overall, and to extract the service time of 500 tickets for probability statistics, in an arbitrary time from this hour, 500 tickets were randomly selected for service time, and according to these 500 The number of tickets sold in a specific time period is shown in the Table 3 below.

Table 3. Number of tickets sold by time period

Peak period		Off-peak period	
Ticketing times	Frequency	Ticketing times	Frequency
0-8	80	0-8	27
8-16	81	8-16	44
16-24	64	16-24	67
24-32	49	24-32	80
32-40	44	32-40	73
40-48	30	40-48	55
48-56	27	48-56	40
56-64	28	56-64	32
64-72	24	64-72	25
72-80	20	72-80	20
80-88	15	80-88	18
>88	38	>88	19

According to the Table 3 above, there is a significant difference in efficiency between peak and off-peak periods. 1. peak periods prompt passengers and ticket agents to work more efficiently, with a substantial increase in efficiency and a significant increase in the speed of ticket issuance.

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2. the significant increase in the number of passengers taking longer than 88min to purchase tickets during peak periods indicates that queuing is common and provides an objective necessity to carry out timely optimisation of the ticket window during peak periods.

A histogram of the distribution of service times based on the service time data for passengers at the ticket window was plotted as follows, and we can view this distribution histogram as a distribution function curve of a continuous random variable.

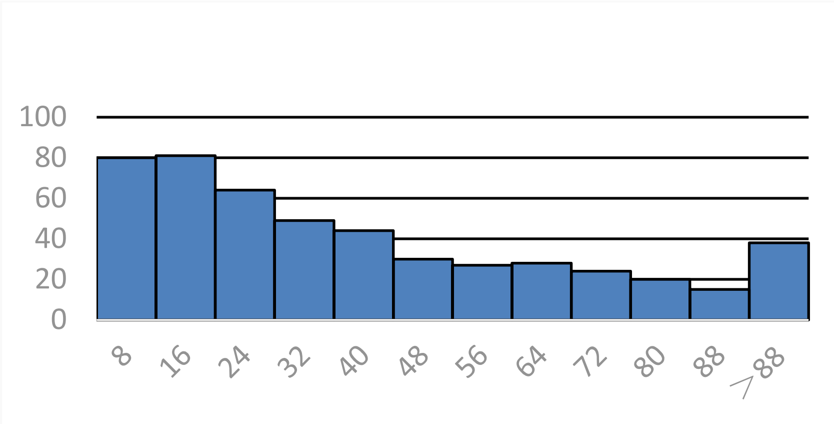

Figure 3. Histogram of the distribution of service hours

Calculate the average service time at the ticket window from the data in the histogram $\bar{t} =$ 36.24s

Calculate the maximum likelihood estimate for the parameter u of the negative exponential distribution with the following formula.

Dividing the histogram into twelve groups of [0,8], [8,16, [16,24], [24,32], [32,40], [40,48], [48,56], [56,64], [64,72], [72,80], [80,88], [88,+∞], the distribution probability of the window service time t is:

$$F(t) = \begin{cases} 1 - e^{-\mu t}, t \geq 0 \\ 0 \quad , t < 0 \end{cases} \tag{6}$$

Calculate the probability $p_i(X = i), p_i = p\{a_{i-1} < x < a_i\} = F(a_i) - F(a_{i-1})$,ai-1 is the lower limit of the interval and ai is the upper limit of the interval.

Before conducting the non-parametric test, we need to determine the original hypothesis H0 and the alternative hypothesis H1 .

H0: The distribution of passenger arrivals is non-complex exponential

H1: The distribution of passenger arrivals is negatively exponential distribution

We used Matlab to write a program to obtain the chi-square values of the test, and as the values of the random variables were divided into 12 groups, the degree of freedom of this test was k-r-1 = 10. By consulting the chi-square distribution table, we learned that the original hypothesis was rejected at the significance level of α = 0.05, so the assumption that the service time at the ticket window obeyed the negative exponential distribution is correct, i.e. the service efficiency of the ticket window during the peak period is μ1 = 1.896 persons/min.

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4.3. Determination of the Optimal Service Window

4.3.1. Maximum Tolerance Time

The results of a random survey of 500 travellers' waiting times for the action of queuing during peak and off-peak periods respectively, based on the frequency of the waiting times, are shown in the Table 4 below.

Table 4. Maximum tolerance time

Peak period		Off-peak period	
Maximum Tolerance Time (Min)	Number of people	Maximum Tolerance Time (Min)	Number of people
(0,5)	70	(0,10)	71
[5,10)	181	[10,15)	168
[10,15)	188	[15,20)	196
[15,+∞)	61	[20,+∞)	65

Calculations from the data in the table above show that the maximum waiting time that passengers can tolerate during peak periods is 15.1 minutes and the maximum waiting time that passengers can tolerate during non-peak periods is 9.9 minutes.

4.3.2. Determination of the Optimal Service Window

With customer arrivals known to be possion flows, with an average arrival rate of λ = 1.9 persons/min, and window service times obeying a negative exponential distribution with an average service time of μ_1 = 1.656 persons/min, now let the passengers arrive in a queue and purchase tickets in turn from the vacant window; this queuing system can be seen as an $M/M/s/\infty$ system.

The simulation simulator is used to simulate the queuing theory system, assuming that the service efficiency of the railway ticketing system is u_1 = 1.2 persons/min during non-peak periods, u_2 = 1.6 persons/min during peak periods, and the average arrival rate of passengers in the ticket hall is in = 5 persons/min. The maximum number of passengers MAXP is infinite, the maximum queue length MAXL is infinite, and the maximum number of service counters MAXS = 7. , the maximum number of service counters MAXS = 7. The upper limit of simulation time MAXT = 100min.

The simulation software was used to simulate the parameters in the previous section under the $M/M/c/\infty$ model to derive the average passenger waiting time at different service efficiencies. This is shown in the Table 5 below.

Table 5. Average passenger waiting time for different service efficiencies

Number of windows	Efficiency of window services	Average passenger waiting time
6	1.2	8.9
	1.6	1.5
7	1.2	2.8
	1.6	0.4

From the simulation results, it can be seen that the average waiting time of passengers is significantly reduced by using high service efficiency at a certain number of ticket windows opening. And at a certain service efficiency, more ticket windows can also save the average waiting time of passengers.

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5. Conclusions and Recommendations

By analysing the results of the simulation tests, it can be concluded that theoretically the more ticket windows opened the better it is to meet the ticketing needs of passengers purchasing tickets during high passenger flows, although there is an upper limit to the number of additional ticket windows that can be opened within the limits of funds allowed by the railway authorities in terms of cost considerations. According to the normal efficiency of window services during peak periods, the maximum customer tolerance time is reached when the number of windows is between 6 and 7, therefore, from the perspective of resource saving, for peak periods, generally speaking, 6-7 windows are open enough to meet all ticketing needs; for off-peak periods, the maximum customer tolerance time is reached when the number of windows is between 3 and 4, so there is no need to open more windows to cause resource There is no need to waste resources.

Therefore, the number of windows should be dynamically adjusted according to the number of passengers purchasing tickets during non-peak periods, and all windows should be opened during peak periods. As the additional windows will be set up under special circumstances, they will be dispersed after the peak period has passed, so the cost to the railways will not be too great a burden. After the overall adjustment, it is possible to optimise some of them, for example by diversifying the number of ticket windows. The windows could be divided into long-haul and short-haul windows, or by province and country, according to the length of the journey. This would be a good way of diverting passengers from buying tickets during peak periods. It is also possible to start with the service staff and provide some training to the ticketing staff at the window to improve the overall business level and service efficiency.

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