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Dynamic Analysis Theory and Application of Cam Mechanism with Elastic Follower

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Abstract

In order to analyze the motion characteristics of the cam mechanism under the consideration of the elastic condition of the follower, this paper theoretically analyzes the actual movement of the moving follower plate cam mechanism under the consideration of the elastic follower, and then analyzes the dynamic analysis of the cam mechanism in the wrapping machine under the consideration of the elasticity of the follower, and the analysis results show that the cam mechanism in the wrapping machine is carried out. (1)the motion amplitude of the actual displacement response changes in comparison with the output motion of the cam theory, and the motion of each period is superimposed on several simple harmonic motions; (2) the amplitude of the vibration of the follower is related to the cam speed. For the dynamic design of the cam mechanism, the rational selection of the cam speed can reduce or even eliminate the vibration. Phenomenon.

Keywords

cam mechanism;elastic follower;kinetics analysis.

1. Foreword

The cam system is widely used in various mechanical equipment, such as textile machines, packaging machines, automatic machine tools, automatic special machine tools, CNC machine tools, printing machines, internal combustion engines, construction machinery, mining machinery, computer auxiliary equipment and agricultural machinery. The dynamic behavior and working performance of the cam system have an important influence on the machine[1].

In the actual structure, the stiffness of the cam itself is often relatively large, and the follower will have a relatively low stiffness due to the distance of the transmitted motion, etc., and because the movement of the follower changes periodically, the instantaneous speed and acceleration of the motion are sometimes It will be very large, and even an impact will occur. In particular, the elasticity of the follower of the high-speed cam mechanism will cause the output motion to be inconsistent with the original design, resulting in uncoordinated mechanical motion, thereby affecting the mechanical performance and reliability of the machine[2]. In addition, due to the effects of periodic loads, elastic vibrations can be caused, which can cause noise and wear of parts. Therefore, to ensure the normal and long-term operation of the cam mechanism, the dynamic analysis of the cam mechanism with elasticity of the follower must be considered.

This paper analyzes and compares the actual motion of the flat cam mechanism with moving followers considering the elastic followers. Finally, the dynamic analysis of the cam mechanism in the wrapper when considering the elastic followers is given.

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2. Establishment of Dynamic Model of Cam Mechanism with Elastic Follower

2.1. Basic Assumptions

When the cam system is used as an elastic mechanical system, some practical conditions in the project are considered, such as high-speed cams are highly designed whether they are designed or manufactured, and the gap between the moving pairs is so small that they can be ignored during actual modeling. The contact deformation is very small, so the contact stiffness can be linearized locally. The complicated damping phenomenon is described by a simple damping mechanism.The actual cam-one follower system is often simplified to be as close as possible to the actual situation. The system of degrees of freedom, and then simplify the multi-degree-of-freedom system into a single-degree-of-freedom system that is easy to analyze and solve.

First make the following assumptions about the cam system: the contour of the cam does not change after the force is applied, there is no kick force in the support, and the angular velocity of the camshaft remains unchanged; in addition, the follower has only axial deformation and the external damping is only considered Viscous damping (usually the damping of metal is very small, this assumption is basically reasonable), the friction of the bearing is not counted; also, further assume that the mass of the system can be expressed as a concentrated mass.

For simplicity and clarity, this article takes a flat cam mechanism with moving followers as an example, as shown in Figure 1, to establish a dynamic model. The dynamic model equations of other forms of cam systems are also similar to the equations of this model.

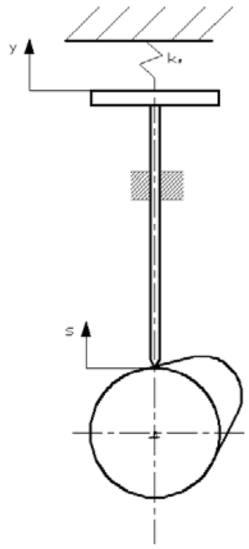

Fig. 1 Simplified simplified model of flat cam follower rod

2.2. Theoretical Analysis

Compared with a transmission system composed of only rotating members around a fixed axis, the link mechanism contains members that perform general planar motion, so motion analysis and dynamic analysis are more complicated. In analyzing the dynamics of the link mechanism with elastic members, the elastic members can be reduced to discrete concentrated mass models, or a finite element model can be established that divides the elastic members into several continuums. In this paper, the follower rod is simplified into a single concentrated mass mechanical model, see Fig.2.

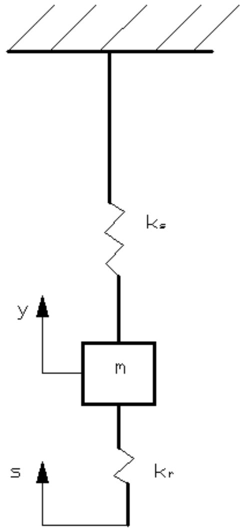

Fig. 2 The dynamic model of the cam mechanism

The equation can be obtained by using the D' Alembert principle[3]:

$$m\ddot{y} + k_r(y - s) + k_s y = 0 \tag{1}$$

Available after simplification:

$$\ddot{y} + \frac{k_r + k_s}{m} y = \frac{k_r}{m} s \tag{2}$$

In the formula:

y---Output motion of follower rod

s---The follower lever obtained by pushing the follower lever from the cam profile bottom movement

m---Driven rod quality

k_r---The equivalent stiffness coefficient of the follower

k_s---The stiffness coefficient of the closed spring of the cam pair.

Assume

$$\omega^2 = \frac{k_r + k_s}{m} \tag{3}$$

$$s = s(\theta) \tag{4}$$

$$\theta = \Omega t \tag{5}$$

In formula (3) (4) (5)

Θ---Cam angle

Ω---The angular velocity of the cam.

Substituting (3) (4) (5) into (2), we can get

$$\ddot{y} + \omega^2 y = \frac{k_r}{m} s(\theta) \tag{6}$$

Equation (6) is the dynamic equation of the cam mechanism in Figure 1. The output motion law of the follower rod when it solves y (t).

According to the assumption, the driven rod is a constant velocity movement, and its movement law is shown in Figure 3.

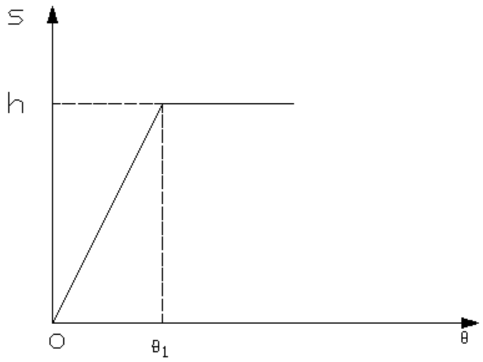

Fig. 3 Regular input of follower rod constant velocity

The regular expression of constant velocity motion is as follows:

$$s(\theta) = \frac{h}{\theta_1}\theta = \frac{h}{\theta_1}\Omega t \tag{7}$$

Substituting equation (7) into equation (6) gives the dynamic equation of constant velocity motion. Its full solution is:

$$y = A\cos\omega t + B\sin\omega t + \frac{k_r}{\omega^2 m}\frac{h}{\theta_1}\Omega t \tag{8}$$

In the formula, A and B are constants, determined by the initial conditions.
When t = 0, y_0 = 0, (y_0)˙ = 0, it can be solved by substituting into (8):

$$A=0, B = -\frac{k_r}{\omega^3 m}\frac{h}{\theta_1}\Omega$$

Therefore:

$$y = -\frac{k_r}{\omega^3 m}\frac{h}{\theta_1}\Omega\sin\omega t + \frac{k_r}{\omega^2 m}\frac{h}{\theta_1}\Omega t \tag{9}$$

If the cam rotation angle is used as the reference coordinate, substitute $\theta = \Omega t$, $\omega^2 = \frac{k_r+k_s}{m}$ into:

$$y(\theta) = \frac{k_r}{k_r + k_s}\left(-\frac{h}{\theta_1}\frac{\Omega}{\omega}\sin\omega\frac{\theta}{\Omega} + \frac{h}{\theta_1}\theta\right) \tag{10}$$

Equation (9) indicates that the mechanism is in the rising stage of the follower, $\theta = 0{\sim}\theta_1$
The law of output motion within the interval. It can be seen from the above formula that during the ascent, the lift is no longer h, and its motion is based on a uniform motion superimposed on a sinusoidal motion. In an interval after $\theta > \theta_1$, the ideal state of the follower is to rest at the highest point. At this time, s (θ) = h, and its dynamic equation is:

$$\ddot{y} + \omega^2 y = \frac{k_r}{m}h \tag{11}$$

The total solution of the equation is:

$$y = A_1\cos\omega t + B_1\sin\omega t + \frac{k_r}{\omega^2 m}h \tag{12}$$

When the initial condition is $\theta = \theta 1$, it can be obtained from equation (10):

$$y(\theta_1) = \frac{k_r}{k_r + k_s}\left(-\frac{h}{\theta_1}\frac{\boldsymbol{\Omega}}{\omega}\sin\boldsymbol{\omega}\frac{\theta_1}{\boldsymbol{\Omega}} + h\right)$$

$$\dot{y}(\theta_1) = \frac{k_r h\boldsymbol{\Omega}}{(k_r + k_s)\theta_1}(-\cos\boldsymbol{\omega}\frac{\theta_1}{\boldsymbol{\Omega}} + 1)$$

Assume $\frac{k_r}{k_r+k_s} = k$, $\omega\frac{\theta_1}{\Omega} = \varphi_1$, $\omega\frac{\theta}{\Omega} = \varphi$, $y(\theta_1) = h_1$, $\dot{y}(\theta_1) = V_1$, Solve equation (12) according to the initial conditions

$$A_1 = (h_1 - kh)cos\varphi_1 - \frac{V_1}{\omega}sin\varphi_1$$

$$B_1 = (h_1 - kh)sin\varphi_1 - \frac{V_1}{\omega}cos\varphi_1$$

Then the output motion in the cam rest interval of in $\theta > \theta_1$ is:

$$\begin{aligned} y &= kh + (h_1 - kh)\cos(\varphi - \varphi_1) + \frac{V_1}{\omega}\sin(\varphi - \varphi_1) \\ &= kh + H\sin(\varphi - \varphi_1 + \alpha) \end{aligned} \tag{13}$$

It can be seen from the above formula that in the "rest" process, the follower superimposes a sinusoidal motion at the position of h.

For the descent process and other motion laws, similar methods can be used to analyze

From the analysis results expressed in equations (10) and (13), it can be seen that the influence of the elasticity of the driven rod on the cam output motion is:

(1)The motion amplitude of the original design changes, and a simple harmonic motion with a circular frequency equal to the natural frequency ω, that is, vibration is superimposed.

(2) The amplitude of the vibration of the follower is related to the ratio of the cam speed Ω and the natural frequency ω. When $\Omega \ll \omega$, each effect is very small. Generally, when $\frac{\Omega}{\omega} = 10^{-2} \sim 10^{-1}$, the influence of the elasticity of the component should be considered.

3. Dynamic Analysis of Cam Mechanism in Wrapping Machine Considering Member Elasticity

The cam mechanism is commonly used in various types of wrapping machines[4]. If the speed of the cam is low and the rigidity of the camshaft and other components is large, the analysis based on the rigid component can obtain more accurate results; when the cam speed is high and the rigidity of the component is small, the elastic effect of the component must be considered[5]. This paper considers the elasticity of the component and studies the dynamic performance of the cam mechanism.

3.1. Cam Mechanism Model

The cam transmission mechanism in the wrapper is shown in Figure 4. In order to ensure that the push rod and the cam are not separated, a spring is installed between the push rod and the box body. The established dynamic model of cam mechanism is shown in Figure 5. The stiffness k_1 is the contact stiffness of the contact surface of the cam and the push rod; k_2 is the tensile stiffness of the push rod; $k_3$is the spring stiffness between the push rod and the box. m_1 and m_2 are the equivalent masses of the putter mass concentrated on both ends according to the principle of constant centroid. u is the theoretical displacement of the cam acting on the follower; y_1 and y_2 are the displacement responses of the two ends of the push rod; the cam performs simple harmonic motion in the thrust and decelerating motion with equal acceleration in the return.

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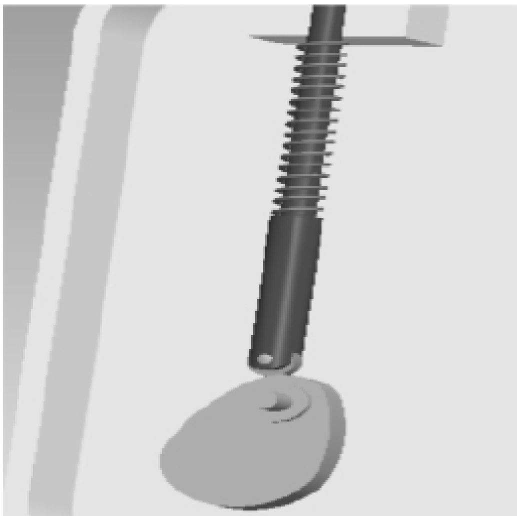

Fig. 4 Schematic diagram of cam mechanism

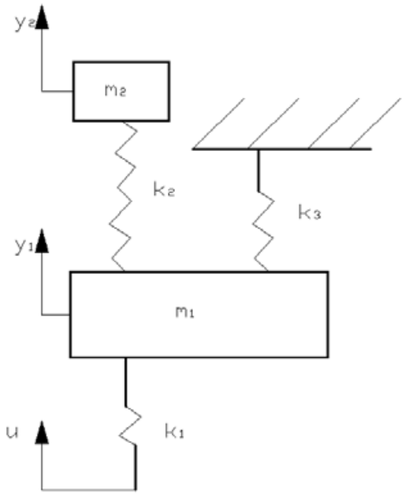

Fig. 5 Dynamic model of cam mechanism

3.2. Kinetics Analysis

The equation can be obtained according to the D' Alembert principle:

$$m_1\ddot{y}_1 + k_2(y_1 - y_2) + k_3 y_1 + k_1(y_1 - u) = 0 \tag{14}$$

$$m_2\ddot{y}_2 - k_2(y_1 - y_2) = 0 \tag{15}$$

u in equation (14) is given by:

$$u = \begin{cases} \frac{h}{2}\left[1 - \cos\left(\frac{\pi}{\delta_t}\omega_1 t\right)\right] & 2\pi n < t \le {(2\pi n + \delta_t)}/{\omega_1} \\ h & {(2\pi n + \delta_t)}/{\omega_1} < t \le {(2\pi n + \delta_t + \delta_s)}/{\omega_1} \\ h - \frac{2h}{{\delta_h}^2}(\omega_1 t - \delta_t - \delta_s)^2 & {(2\pi n + \delta_t + \delta_s)}/{\omega_1} < t \le {(2\pi n + \delta_t + \delta_s + \delta_s')}/{\omega_1} \\ \frac{2h}{{\delta_h}^2}(\delta_t + \delta_s + \delta_h - \omega_1 t)^2 & {(2\pi n + \delta_t + \delta_s + \delta_s')}/{\omega_1} < t \le {(2\pi n + \delta_t + \delta_s + \delta_h)}/{\omega_1} \\ 0 & {(2\pi n + \delta_t + \delta_s + \delta_h)}/{\omega_1} < t \le {(2\pi n + 2\pi)}/{\omega_1} \end{cases} \tag{16}$$

Where h designates the motion amplitude; ω_1 is the cam speed; δ_t δ_s δ_h $\delta_s{}'$,the angle of rotation of the driven rod at different stages of movement Where n takes a non-negative integer.

From the formula (14) (15) available:

$$y_2{}^{(4)} + \frac{m_1 k_2 + m_2(k_1 + k_2 + k_3)}{m_1 m_2}\ddot{y}_2 + \frac{k_2(k_1 + k_3)}{m_1 m_2}y_2 = \frac{k_1 k_2}{m_1 m_2}u \tag{17}$$

Assume$a = \frac{m_1 k_2 + m_2(k_1 + k_2 + k_3)}{m_1 m_2}$, $b = \frac{k_2(k_1 + k_3)}{m_1 m_2}$, $c = \frac{k_1 k_2}{m_1 m_2}$,Then the differential equation makes the general solution can be expressed as:

$$y = A_1 \cos\sqrt{\frac{a}{2} - \sqrt{\frac{a^2}{4} - b}}\,t + B_1 \sin\sqrt{\frac{a}{2} - \sqrt{\frac{a^2}{4} - b}}\,t + C_1 \cos\sqrt{\frac{a}{2} + \sqrt{\frac{a^2}{4} - b}}\,t + D_1 \sin\sqrt{\frac{a}{2} + \sqrt{\frac{a^2}{4} - b}}\,t \tag{18}$$

The A_1 B_1 C_1 D_1 in the above formula are constants, determined by the initial conditions.

When$2\pi n < t \le {(2\pi n + \delta_t)}/{\omega_1}$, cam theoretical output movement$u = \frac{h}{2}\left[1 - \cos\left(\frac{\pi}{\delta_t}\omega_1 t\right)\right]$,

The special solution corresponding to equation(17):

$$y^*_1 = -\frac{h}{2}\frac{c}{\left(\frac{\pi}{\delta_t}\omega_1\right)^4 - a\left(\frac{\pi}{\delta_t}\omega_1\right)^2 + b}\cos\left(\frac{\pi}{\delta_t}\omega_1 t\right) + \frac{hc}{2b} \tag{19}$$

At this time, the general solution of equation (17) is:

$$y(\mathrm{t}) = y + y^*_1 \tag{20}$$

In the same way, we can get the special solutions of the differential equations for the rest of the time, respectively:

$$y^*_2 = \frac{hc}{b} \tag{21}$$

$$y^*_3 = -\frac{2h{\omega_1}^2}{{\delta_h}^2}\frac{c}{b}t^2 + \frac{4h\omega_1}{{\delta_h}^2}(\delta_t+\delta_s)\frac{c}{b}t + \frac{c}{b}h - \frac{2h}{{\delta_h}^2}(\delta_t+\delta_s)^2\frac{c}{b} + \frac{4h{\omega_1}^2}{{\delta_h}^2}\frac{ac}{b^2} \tag{22}$$

$$y^*_4 = \frac{2h{\omega_1}^2}{{\delta_h}^2}\frac{c}{b}t^2 - \frac{4h\omega_1}{{\delta_h}^2}(\delta_t+\delta_s+\delta_h)\frac{c}{b}t + \frac{2h}{{\delta_h}^2}(\delta_t+\delta_s+\delta_h)^2\frac{c}{b} - \frac{4h{\omega_1}^2}{{\delta_h}^2}\frac{ac}{b^2} \tag{23}$$

$$y^*_5 = 0 \tag{24}$$

Therefore, when $2\pi n < t \leq {(2\pi n + \delta_t)}/{\omega_1}$, the actual displacement response is:

$$y = A_1\cos\sqrt{\frac{a}{2} - \sqrt{\frac{a^2}{4} - b}}\,t + B_1\sin\sqrt{\frac{a}{2} - \sqrt{\frac{a^2}{4} - b}}\,t + C_1\cos\sqrt{\frac{a}{2} + \sqrt{\frac{a^2}{4} - b}}\,t + D_1\sin\sqrt{\frac{a}{2} + \sqrt{\frac{a^2}{4} - b}}\,t - \frac{h}{2}\frac{c}{\left(\frac{\pi}{\delta_t}\omega_1\right)^4 - a\left(\frac{\pi}{\delta_t}\omega_1\right)^2 + b}\cos\left(\frac{\pi}{\delta_t}\omega_1 t\right) + \frac{hc}{2b} \tag{25}$$

The A_1 B_1 C_1 D_1 in the above formula are constants, determined by the initial conditions.

When ${(2\pi n + \delta_t)}/{\omega_1} < t \leq {(2\pi n + \delta_t + \delta_s)}/{\omega_1}$, the actual displacement response is:

$$y = A_2\cos\sqrt{\frac{a}{2} - \sqrt{\frac{a^2}{4} - b}}\,t + B_2\sin\sqrt{\frac{a}{2} - \sqrt{\frac{a^2}{4} - b}}\,t + C_2\cos\sqrt{\frac{a}{2} + \sqrt{\frac{a^2}{4} - b}}\,t + D_2\sin\sqrt{\frac{a}{2} + \sqrt{\frac{a^2}{4} - b}}\,t + \frac{hc}{b} \tag{26}$$

The A_2 B_2 C_2 D_2 in the above formula are constants, determined by the initial conditions.

When ${(2\pi n + \delta_t + \delta_s)}/{\omega_1} < t \leq {(2\pi n + \delta_t + \delta_s + \delta_s')}/{\omega_1}$,the actual displacement response is:

$$y = A_3\cos\sqrt{\frac{a}{2} - \sqrt{\frac{a^2}{4} - b}}\,t + B_3\sin\sqrt{\frac{a}{2} - \sqrt{\frac{a^2}{4} - b}}\,t + C_3\cos\sqrt{\frac{a}{2} + \sqrt{\frac{a^2}{4} - b}}\,t + D_3\sin\sqrt{\frac{a}{2} + \sqrt{\frac{a^2}{4} - b}}\,t - \frac{2h{\omega_1}^2}{{\delta_h}^2}\frac{c}{b}t^2 + \frac{4h\omega_1}{{\delta_h}^2}(\delta_t+\delta_s)\frac{c}{b}t + \frac{c}{b}h - \frac{2h}{{\delta_h}^2}(\delta_t+\delta_s)^2\frac{c}{b} + \frac{4h{\omega_1}^2}{{\delta_h}^2}\frac{ac}{b^2} \tag{27}$$

The $A_3 B_3$ C_3 D_3 in the above formula are constants, determined by the initial conditions.

When $(2\pi n+\delta_t+\delta_s+\delta_s')/_{\omega_1} < t \le (2\pi n+\delta_t+\delta_s+\delta_h)/_{\omega_1}$ the actual displacement response is:

$$y = A_4 cos\sqrt{\frac{a}{2}-\sqrt{\frac{a^2}{4}-b}}\,t + B_4 sin\sqrt{\frac{a}{2}-\sqrt{\frac{a^2}{4}-b}}\,t + C_4 cos\sqrt{\frac{a}{2}+\sqrt{\frac{a^2}{4}-b}}\,t + D_4 sin\sqrt{\frac{a}{2}+\sqrt{\frac{a^2}{4}-b}}\,t + \frac{2h\omega_1^2}{\delta_h^2}\frac{c}{b}t^2 - \frac{4h\omega_1}{\delta_h^2}(\delta_t+\delta_s+\delta_h)\frac{c}{b}t + \frac{2h}{\delta_h^2}(\delta_t+\delta_s+\delta_h)^2\frac{c}{b} - \frac{4h\omega_1^2}{\delta_h^2}\frac{ac}{b^2} \tag{28}$$

The $A_4 B_4$ C_4 D_4 in the above formula are constants, determined by the initial conditions.
When $(2\pi n+\delta_t+\delta_s+\delta_h)/_{\omega_1} < t \le (2\pi n+2\pi)/_{\omega_1}$, the actual displacement response is:

$$y = A_5 cos\sqrt{\frac{a}{2}-\sqrt{\frac{a^2}{4}-b}}\,t + B_5 sin\sqrt{\frac{a}{2}-\sqrt{\frac{a^2}{4}-b}}\,t + C_5 cos\sqrt{\frac{a}{2}+\sqrt{\frac{a^2}{4}-b}}\,t + D_5 sin\sqrt{\frac{a}{2}+\sqrt{\frac{a^2}{4}-b}}\,t \tag{29}$$

The $A_5 B_5$ C_5 D_5 in the above formula are constants, determined by the initial conditions.

4. Conclusion

After considering the influence of the elasticity of the follower on the output motion of the cam, comparing the theoretical output motion of the cam of formula (16) with the actual displacement response of formula (25) (26) (27) (28) (29), the following conclusion is drawn :
(1) The motion amplitude of the actual displacement response has changed compared to the theoretical output motion of the cam, and the motion of each period is superimposed with several simple harmonic motions;
(2) The amplitude of the vibration of the follower is related to the cam speed. For the dynamic design of the cam mechanism, a reasonable selection of the cam speed can reduce or even eliminate the vibration phenomenon.
Therefore, if the elastic effect of the rod is not considered, the fatigue life of the cam mechanism cannot be accurately obtained.

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